Eur. Phys. J. C 41, 233–243 (2005)
Digital Object Identifier (DOI) 10.1140/epjc/s2005-02174-4

THE EUROPEAN
PHYSICAL JOURNAL C

Lepton flavor violating $Z \to l_1^+ l_2^-$ decay in the two Higgs doublet model with the inclusion of non-universal extra dimensions

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Received: 5 November 2004 / Revised version: 29 January 2005 /
Published online: 16 March 2005 –

Abstract. We predict the branching ratios of the $Z \to e^\pm \mu^\pm$, $Z \to e^\pm \tau^\pm$ and $Z \to \mu^\pm \tau^\pm$ decays in the model III version of the two Higgs doublet model, with the inclusion of one and two spatial non-universal extra dimensions. We observe that the branching ratios are not sensitive to a single extra dimension, however, this sensitivity is considerable for two extra dimensions

1 Introduction

The lepton flavor violating (LFV) interactions are interesting in the sense that they are sensitive to physics beyond the standard model (SM) and they ensure that one can obtain considerable information about the restrictions of the free parameters, appearing in the new models, with the help of the possible accurate measurements. Among LFV interactions, the Z decays with different lepton flavor outputs, such as $Z \to e\mu$, $Z \to e\tau$ and $Z \to \mu\tau$, are rich enough to study and there is an extensive work related to these decays in the literature [1–12]. The Giga-Z option of the Tesla project which aims to increase the production of Z bosons at resonance [13] stimulates one to make theoretical works on such Z decays.

In the framework of the SM the lepton flavor is conserved and, for the flavor violation in the lepton sector, there is a need to extend the SM. One of the candidate models is the so called νSM, which is constructed by taking the neutrinos massive and permitting the lepton mixing mechanism [14]. In this model, the theoretical predictions for the branching ratios (BRs) of the LFV Z decays are extremely small in the case of internal light neutrinos [1,2]:

$$\begin{aligned} &\mathrm{BR}(Z \to e^\pm \mu^\pm) \sim \mathrm{BR}(Z \to e^\pm \tau^\pm) \sim 10^{-54}\,, \\ &\mathrm{BR}(Z \to \mu^\pm \tau^\pm) < 4 \times 10^{-60}. \end{aligned} \tag{1}$$

They are far from the experimental limits obtained at LEP 1 [15]:

$$\mathrm{BR}(Z \to e^\pm \mu^\pm) < 1.7 \times 10^{-6} \;\; [3]\,,$$
$$\mathrm{BR}(Z \to e^\pm \tau^\pm) < 9.8 \times 10^{-6} \;\; [3,4]\,, \tag{2}$$
$$\mathrm{BR}(Z \to \mu^\pm \tau^\pm) < 1.2 \times 10^{-5} \;\; [3,5] \tag{3}$$

and from the improved ones at Giga-Z [6]:

$$\mathrm{BR}(Z \to e^\pm \mu^\pm) < 2 \times 10^{-9}\,,$$
$$\begin{aligned} &\mathrm{BR}(Z \to e^\pm \tau^\pm) < f \times 6.5 \times 10^{-8}\,, \\ &\mathrm{BR}(Z \to \mu^\pm \tau^\pm) < f \times 2.2 \times 10^{-8}, \end{aligned} \tag{4}$$

with $f = 0.2$–1.0. Notice that these numbers are obtained for the decays $Z \to \bar{l}_1 l_2 + \bar{l}_2 l_1$, namely

$$\mathrm{BR}(Z \to l_1^\pm l_2^\pm) = \frac{\Gamma(Z \to \bar{l}_1 l_2 + \bar{l}_2 l_1)}{\Gamma_Z}\,. \tag{5}$$

To enhance the BRs of the corresponding LFV Z decays some other scenarios have been studied. The possible scenarios are the extension of νSM with one heavy ordinary Dirac neutrino [2], the extension of νSM with two heavy right-handed singlet Majorana neutrinos [2], the Zee model [7], the model III version of the two Higgs doublet model (2HDM), which is the minimal extension of the SM [8], the supersymmetric models [9,10], and the top–color assisted technicolor model [11].

The present work is devoted to predictions of the BRs of the $Z \to e^\pm \mu^\pm$, $Z \to e^\pm \tau^\pm$ and $Z \to \mu^\pm \tau^\pm$ decays in the model III version of the 2HDM, with the inclusion of one and two spatial extra dimensions. Our motivation is to check whether there is an enhancement in the BRs of these decays due to the extra dimensions. The possible existence of new dimensions gained great interest recently and there is a large amount of work done in the literature [16–32]. The idea of extra dimensions originated from the study of Kaluza–Klein [33] which was related to the unification of electromagnetism and gravity and the motivation increased with the study of string theory which was formulated in a space-time of more than four dimensions. Since the extra dimensions are hidden to the experiments at present (for example see [30]), the most favorable description is that these new dimensions are compactified to the surfaces with small radii, which is a typical size of the corresponding extra dimension. This leads to the appearance of new particles, namely Kaluza–Klein (KK) modes of the particles in the theory. In the case that all the fields feel

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the extra dimensions, so called universal extra dimensions (UED), the extra dimensional momentum, and therefore, the KK number at each vertex is conserved. The compactification size R has been predicted to be as large as a few hundreds of GeV [17–20], in the range 200–500 GeV, using electroweak precision measurements [21], B–$\bar{B}$ mixing [22,23] and the flavor changing process $b \to s\gamma$ [24] for a single UED.

Under the assumption that the extra dimensions are of the order of the submillimeter distance, for two extra dimensions, the hierarchy problem in the fundamental scales could be solved and the true scale of quantum gravity would be no more the Planck scale but of the order of the electroweak (EW) scale [16,17]. In this case, gravity is spreading over all the volume including the extra dimensions; however, the matter fields are restricted in four dimensions, called four dimensional (4D) brane, or in a 4D surface which has a non-zero thickness in the new dimensions, called a fat brane (see for example [25]). This type of extra dimensions, accessible to some fields but not all in the theory, are called non-universal extra dimensions (NUED). Contrary to the UED, in the NUED, the KK number at each vertex is not conserved and a tree level interaction of KK modes with the ordinary particles can exist. The study in [26] is devoted to the appearance of a very light left-handed neutrino in the NUED where only the right-handed neutrino is accessible to the extra dimension. In another work [27], the effect of brane kinetic terms for bulk scalars, fermions and gauge fields in higher dimensional theories have been studied. In [28] the electric dipole moments of fermions and some LFV decays have been analyzed in the framework of NUED.

Here, we predict the BRs of the LFV Z decays in model III with the assumption that the extra dimensions are felt by the new Higgs doublet and the gauge sector of the theory. The Z decays under consideration are induced by the internal neutral Higgs bosons h^0 and A^0, and their KK modes carry all the information about the new dimensions, after the compactification of the single (double) extra dimension on the orbifold S^1/Z_2 $((S^1 \times S^1)/Z_2)$. Here, the KK number in the vertices is not conserved, in contrast to the UED case. The non-zero KK modes of neutral Higgs fields H have masses $\sqrt{m_H^2 + m_n^2}$ $(\sqrt{m_H^2 + m_n^2 + m_r^2})$ with $m_k = k/R$ in one (two) extra dimension(s). We observe that the BRs of the processes we study are enhanced to almost two orders larger compared to the ones without the extra dimensions, in the case of two NUED, since the neutral Higgs KK modes are considerably crowded.

This paper is organized as follows: In Sect. 2, we present the effective vertex and the BRs of LFV Z decays in the model III version of the 2HDM with the inclusion of NUED. Section 3 is devoted to a discussion and our conclusions. In the appendix section, we give the explicit expressions of the form factors appearing in the effective vertex.

2 $Z \to l_1^- l_2^+$ decay in model III with the inclusion of non-universal extra dimensions

The extension of the Higgs sector in the SM brings new contributions to the BRs of the processes and makes it possible to obtain the flavor changing neutral current (FCNC) at tree level, which plays an important role in the existence of flavor violating (FV) interactions. Therefore, the multi-Higgs doublet models are worthwhile to study. The 2HDM is one of the candidates for the multi-Higgs doublet models. In the model I and II versions of the 2HDM, the FCNC at tree level is forbidden, however, those types of interactions are possible in the model III version of the 2HDM. The lepton flavor violating (LFV) Z decay $Z \to l_1^- l_2^+$ can be induced at least in the one loop level in the framework of model III.

The addition of possible NUED, which are experienced by the gauge bosons and the new Higgs particles, brings new contributions to the BRs of the decays under consideration. In the model III, the part of the Lagrangian which carries the interaction, responsible for the LFV processes in 5 (6) dimensions, reads

$$\mathcal{L}_Y = \xi^D_{5(6)\,ij} \bar{l}_i \,(\phi_2|_{y(z)=0})\, E_j + \text{h.c.} \;, \tag{6}$$

where the couplings $\xi^D_{5(6)\,ij}$ are 5(6)-dimensional dimensionful Yukawa couplings which induce the LFV interactions. These couplings can be rescaled to the ones in four dimensions as $\xi^D_{5(6)\,ij} = \sqrt{2\pi R}\,(2\pi R)\,\xi^D_{ij}$ with lepton family indices i, j[1]. Here, ϕ_2 is the new scalar doublet, R is the compactification radius, l_i and E_j are lepton doublets and singlets, respectively. The scalar and lepton doublets are functions of x^μ and y (y, z), where y (z) is the coordinate representing the 5(6)th dimension. Here we assume that the Higgs doublet lying in the 4-dimensional brane has a non-zero vacuum expectation value to ensure the ordinary masses of the gauge fields and the fermions, however, the second doublet, which is accessible to the extra dimensions, has no vacuum expectation value, namely, we choose the doublets ϕ_1 and ϕ_2 and their vacuum expectation values as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ \mathrm{i}\chi^0 \end{pmatrix} \right] ;$$
$$\phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ H_1 + \mathrm{i} H_2 \end{pmatrix} , \tag{7}$$

and

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \; ; \quad \langle \phi_2 \rangle = 0 \; . \tag{8}$$

This choice ensures that the mixing between neutral scalar Higgs bosons is switched off and it would be possible to separate the particle spectrum so that the SM particles are collected in the first doublet and the new particles in the second one[2]. Here we consider the gauge and CP

[1] In the following, we replace ξ^D with ξ^D_{N} where "N" denotes the word "neutral".

[2] Here H^1 (H^2) is the well known mass eigenstate h^0 (A^0).

invariant Higgs potential in two extra dimensions:

$$
\begin{aligned}
&V(\phi_1, \phi_2) \\
&= \delta(y)\,\delta(z)\,c_1\,(\phi_1^+\phi_1 - v^2/2)^2 + c_2\,(\phi_2^+\phi_2)^2 \\
&+ \delta(y)\,\delta(z)\,\big(c_3[(\phi_1^+\phi_1 - v^2/2)\,(\phi_2^+\phi_2)] \\
&+ c_4[(\phi_1^+\phi_1)(\phi_2^+\phi_2) - (\phi_1^+\phi_2)(\phi_2^+\phi_1)] \\
&+ \; c_5[\mathrm{Re}(\phi_1^+\phi_2)]^2 + c_6[\mathrm{Im}(\phi_1^+\phi_2)]^2\big) + c_7\,, \qquad (9)
\end{aligned}
$$

with constants c_i, $i = 1, ..., 7$.

Since only the new Higgs field ϕ_2 is accessible to extra dimensions in the Higgs sector, there appear KK modes $\phi_2^{(n,r)}$ of ϕ_2 in two spatial extra dimensions after the compactification of the external dimensions on an orbifold $(S^1 \times S^1)/Z_2$,

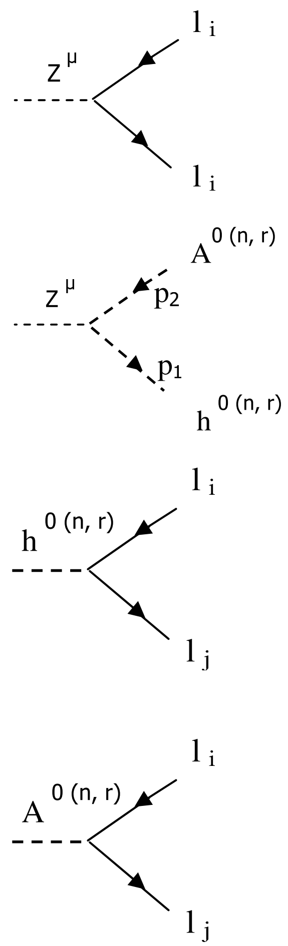
$$
\begin{aligned}
\phi_2(x, y, z) &= \frac{1}{(2\pi R)^{d/2}} \qquad (10) \\
&\times \left\{ \phi_2^{(0,0)}(x) + 2^{d/2} \sum_{n,r}^{\infty} \phi_2^{(n,r)}(x) \cos(ny/R + rz/R) \right\},
\end{aligned}
$$

where $d = 2$; the indices n and r are positive integers including zero, but both are not zero at the same time. Here, $\phi_2^{(0,0)}(x)$ is the 4-dimensional Higgs doublet which includes the charged Higgs boson H^+, the neutral CP even (odd) Higgs bosons h^0 (A^0). The KK modes of the charged Higgs boson (neutral CP even (odd) Higgs h^0 (A^0)) have the mass $\sqrt{m_{H^\pm}^2 + m_n^2 + m_r^2}$ $(\sqrt{m_{h^0}^2 + m_n^2 + m_r^2}$ $(\sqrt{m_{A^0}^2 + m_n^2 + m_r^2}))$, where $m_n = n/R$ and $m_r = r/R$. Furthermore, we assume that the compactification radius R is the same for both new dimensions. Notice that the expansion for a single extra dimension can be obtained by setting $d = 1$, taking $z = 0$, and dropping the summation over r. In addition to the new Higgs doublet, also the gauge fields feel the extra dimensions, however, there is no additional contribution coming from the KK modes of the Z boson in the process under consideration since the Z boson does not enter in the calculations as an internal line. The Z–h^0 KK mode–A^0 KK mode vertex is the same as the 4-dimensional one after integration over the extra dimensions, except for a small correction of the coupling due to the gauge field 0–mode–KK mode mixing (see the appendix, Sect. **??**, for details).

Now, we would like to present the general effective vertex for the interaction of an on-shell Z boson with a fermionic current:

$$
\Gamma_\mu = \gamma_\mu(f_V - f_A\,\gamma_5) + \frac{\mathrm{i}}{m_W}\,(f_M + f_E\,\gamma_5)\,\sigma_{\mu\,\nu}\,q^\nu, \qquad (11)
$$

where q is the momentum transfer, $q^2 = (p - p')^2$, f_V (f_A) is vector (axial-vector) coupling, and f_M (f_E) the magnetic (electric) transitions of unlike fermions. Here p $(-p')$ is the four momentum vector of the lepton (antilepton) (see Fig. 2 for the necessary one loop diagrams due to neutral Higgs particles). Since LFV Z boson decay exists at least on the loop level, the KK modes of neutral



$$\frac{-\mathrm{i}\,g_Z}{c_W}\,\gamma^\mu\,[c_L\,L + c_R\,R]$$

$$\frac{g_Z}{2\,c_W}\,(p_2 + p_1)^\mu$$

$$\frac{-\mathrm{i}}{2\sqrt{2}}\left[(\xi_{ij}^D + \xi_{ji}^{D*}) + (\xi_{ij}^D - \xi_{ji}^{D*})\gamma_5\right]$$

$$\frac{1}{2\sqrt{2}}\left[(\xi_{ij}^D - \xi_{ji}^{D*}) + (\xi_{ij}^D + \xi_{ji}^{D*})\gamma_5\right]$$

Fig. 1. The vertices used in the present work. Here L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, the parameters $c_{L\,(R)}$ read $c_L = -1/2 + s_W^2$, $c_R = s_W^2$ and $c_W = \cos\theta_W$, $s_W = \sin\theta_W$, where θ_W is the weak angle. Notice that the tree level interaction Z^μ–H^0–A^0 does not exist, since since there is no mixing between the neutral scalar Higgs bosons H^0 and h^0 due to our choice

Higgs particles h^0 and A^0 contribute to the self-energy and vertex diagrams as internal lines. The leptons live in the 4D brane and therefore they do not have any KK modes. Notice that, in the case of a non-universal extra dimension, the KK number needs not to be conserved and there exist lepton–lepton–h^0 KK mode (A^0 KK mode) vertices which can involve two zero modes and one KK mode.

The vector (axial-vector) f_V (f_A) couplings and the magnetic (electric) transitions f_M (f_E) including the contributions coming from a single extra dimension can be obtained as

$$
\begin{aligned}
f_V &= f_V^{(0)} + 2\sum_{n=1}^{\infty} f_V^{(n)}\,, \\
f_A &= f_A^{(0)} + 2\sum_{n=1}^{\infty} f_A^{(n)}\,,
\end{aligned}
$$

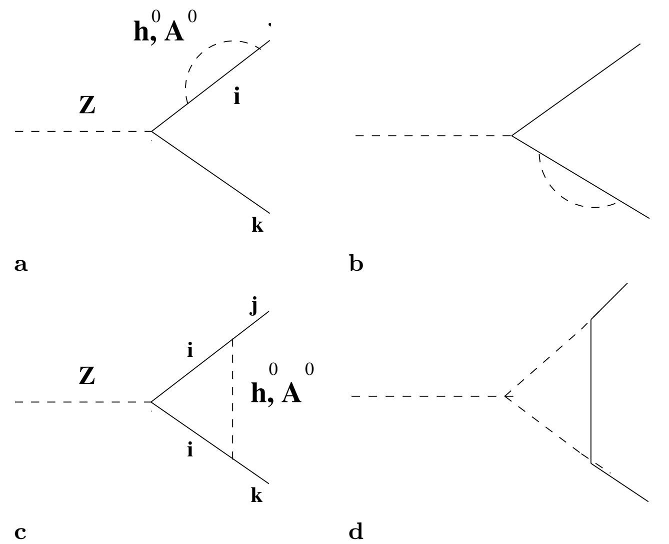


Fig. 2. One loop diagrams contribute to $Z \to k^+ j^-$ decay due to the neutral Higgs bosons h_0 and A_0 in the 2HDM. i represents the internal, j (k) the outgoing (incoming) lepton, dashed lines the vector field Z, and the h_0 and A_0 fields. In the case of 5 (6) dimensions the vertices are the same but there are additional contributions due to the KK modes of the h_0 and A_0 fields

$$
\begin{aligned}
f_M &= f_M^{(0)} + 2 \sum_{n=1}^{\infty} f_M^{(n)} \, , \\
f_E &= f_E^{(0)} + 2 \sum_{n=1}^{\infty} f_E^{(n)} \, ,
\end{aligned}
\tag{12}
$$

where $f_{V,A,M,E}^{(0)}$ are the couplings in the four dimensions and $f_{V,A,M,E}^{(n)}$ are the ones due to the KK modes of the scalar bosons $S = h^0, A^0$. The KK mode contributions $f_{V,A,M,E}^{(n)}$ can be easily obtained by replacing the mass squares m_S^2 in $f_{V,A,M,E}^{(0)}$ by $m_S^2 + m_n^2$, with $m_n = n/R$ and the compactification radius R. We present the explicit expressions for the couplings $f_{V,A,M,E}^{(0)}$ in the appendix, by taking into account all the masses of the internal leptons and external lepton (anti-lepton).

If we consider two NUED, the couplings $f_{V,A,M,E}^{(n)}$ appearing in (12) should be replaced by $f_{V,A,M,E}^{(n,r)}$ and the summation would be done over $n, r = 0, 1, 2, ...$ except for $n = r = 0$. Here $f_{V,A,M,E}^{(n,r)}$ can be obtained by replacing the mass squares m_S^2 in $f_{V,A,M,E}^{(0)}$ by $m_S^2 + m_n^2 + m_r^2$, with $m_n = n/R$, $m_r = r/R$. Furthermore, the number 2 in front of the summations in (12) would be replaced by 4.

Finally, the BR for $Z \to l_1^- \, l_2^+$ can be written in terms of the couplings f_V, f_A, f_M and f_E as

$$
\begin{aligned}
\mathrm{BR}(Z \to l_1^- \, l_2^+) &= \frac{1}{48\,\pi} \frac{m_Z}{\Gamma_Z} \\
&\times \left\{ |f_V|^2 + |f_A|^2 + \frac{1}{2\cos^2\theta_{\mathrm{W}}} (|f_M|^2 + |f_E|^2) \right\} ,
\end{aligned}
\tag{13}
$$

where $\alpha_W = \frac{g^2}{4\,\pi}$ and Γ_Z is the total decay width of the Z boson. In our numerical analysis we consider the BR due to the production of a sum of charged states, namely

$$
\mathrm{BR}(Z \to l_1^{\pm} \, l_2^{\pm}) = \frac{\Gamma(Z \to (\bar{l}_1 \, l_2 + \bar{l}_2 \, l_1)}{\Gamma_Z} \, . \tag{14}
$$

3 Discussion

The LFV Z decays $Z \to e^{\pm}\mu^{\pm}$, $Z \to e^{\pm}\tau^{\pm}$ and $Z \to \mu^{\pm}\tau^{\pm}$ strongly depend on the Yukawa couplings $\bar{\xi}^D_{\mathrm{N},ij}$ [3], $i, j = e, \mu, \tau$ in the model III version of 2HDM and these couplings are free parameters which should be restricted by using the present and forthcoming experiments. At first, we assume that the couplings which contain at least one τ index are dominant similar to the Cheng–Sher scenario [34] and, therefore, we consider only the internal τ lepton case among others. Furthermore, we assume that the Yukawa couplings $\bar{\xi}^D_{\mathrm{N},ij}$ are symmetric with respect to the indices i and j. As a result, we need the numerical values for the couplings $\bar{\xi}^D_{\mathrm{N},\tau e}$, $\bar{\xi}^D_{\mathrm{N},\tau\mu}$ and $\bar{\xi}^D_{\mathrm{N},\tau\tau}$.

The upper limit of $\bar{\xi}^D_{\mathrm{N},\tau\mu}$ is predicted as $30\,\mathrm{GeV}$ (see [35] and references therein) by using the experimental uncertainty, 10^{-9}, in the measurement of the muon anomalous magnetic moment and assuming that the new physics effects cannot exceed this uncertainty. Using this upper limit and the experimental upper bound of the BR of the $\mu \to e\gamma$ decay, $\mathrm{BR} \leq 1.2 \times 10^{-11}$, the coupling $\bar{\xi}^D_{\mathrm{N},\tau e}$ can be restricted in the range, 10^{-3}–$10^{-2}\,\mathrm{GeV}$ [36]. For the Yukawa coupling $\bar{\xi}^D_{\mathrm{N},\tau\tau}$, we have no explicit restriction region and we use the numerical values which are greater than $\bar{\xi}^D_{\mathrm{N},\tau\mu}$. Furthermore, the addition of the extra dimensions bring us a new parameter, namely the compactification radius R which arises from the compactification of a single (double) extra dimension on the orbifold S^1/Z_2 ($(S^1 \times S^1)/Z_2$).

In the present work, we study the prediction of the NUED on the BR of the LFV processes $Z \to l_1^{\pm} l_2^{\pm}$, in the framework of type III 2HDM. We see that the contributions coming from two extra dimensions are considerably large compared to the one coming from a single extra dimension, due to the crowd of neutral scalar Higgs boson KK modes.

Throughout our calculations we use the input values given in Table 1.

Figure 3 is devoted to the $\bar{\xi}^D_{\mathrm{N},\tau e}$ dependence of $\mathrm{BR}(Z \to \mu^{\pm}\, e^{\pm})$ for $\bar{\xi}^D_{\mathrm{N},\tau\mu} = 1\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$ and $m_{A^0} = 200\,\mathrm{GeV}$. The solid, dashed, and small dashed lines represent the BR without extra dimension, including a single extra dimension for $1/R = 500\,\mathrm{GeV}$, and including two extra dimensions for $1/R = 500\,\mathrm{GeV}$. It is observed that the BR is not sensitive to the extra dimension effects for a single extra dimension. However, for two NUED, there is a considerable enhancement, of almost two orders, in the BR compared to the one without extra dimensions,

[3] The dimensionful Yukawa couplings $\bar{\xi}^D_{\mathrm{N},ij}$ are defined as $\xi^E_{\mathrm{N},ij} = \sqrt{\frac{4\,G_{\mathrm{F}}}{\sqrt{2}}}\, \bar{\xi}^D_{\mathrm{N},ij}$.

Table 1. The values of the input parameters used in the numerical calculations

Parameter	Value
m_μ	0.106 (GeV)
m_τ	1.78 (GeV)
m_W	80.26 (GeV)
m_Z	91.19 (GeV)
G_{F}	$1.16637 10^{-5}$ (GeV^{-2})
Γ_Z	2.490 (GeV)
$\sin\,\theta_{\mathrm{W}}$	$\sqrt{0.2325}$

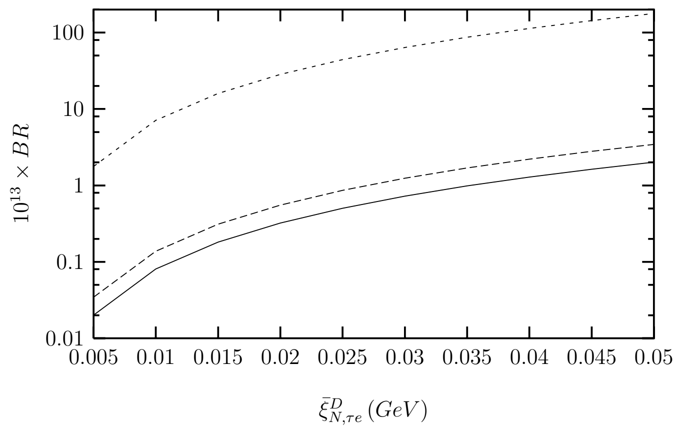


Fig. 3. $\bar{\xi}^{D}_{\mathrm{N},\tau e}$ dependence of $\mathrm{BR}\,(Z \to \mu^\pm e^\pm)$ for $\bar{\xi}^{D}_{\mathrm{N},\tau\mu} = 1\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$ and $m_{A^0} = 200\,\mathrm{GeV}$. The solid, dashed and small dashed lines represent the BR without extra dimension, including a single extra dimension for $1/R = 500\,\mathrm{GeV}$ and including two extra dimensions for $1/R = 500\,\mathrm{GeV}$

even for the small values of the coupling $\bar{\xi}^{D}_{\mathrm{N},\tau\mu}$. This is due to the crowd of neutral Higgs boson KK modes. This enhancement can be observed also in Fig. 4 where the BR is plotted with respect to the compactification scale $1/R$ for $\bar{\xi}^{D}_{\mathrm{N},\tau e} = 0.05\,\mathrm{GeV}$, $\bar{\xi}^{D}_{\mathrm{N},\tau\mu} = 1\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$ and $m_{A^0} = 200\,\mathrm{GeV}$. In this figure the solid, dashed and small dashed lines represent the BR without extra dimension, including a single extra dimension and including two extra dimensions. It is seen that in the case of two extra dimensions BR reaches almost twice the one without extra dimensions, for the values of the compactification scale, $1/R = 2000\,\mathrm{GeV}$. This enhancement becomes negligible for the larger values of the compactification scales, $1/R > 5000\,\mathrm{GeV}$. The possible enhancement due to the effect of two NUED on the theoretical value of the BR of the corresponding Z decay is worthwhile to study.

In Fig. 5, we present the $\bar{\xi}^{D}_{\mathrm{N},\tau\tau}$ dependence of the $\mathrm{BR}\,(Z \to \tau^\pm e^\pm)$ for $\bar{\xi}^{D}_{\mathrm{N},\tau e} = 0.05\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$ and $m_{A^0} = 200\,\mathrm{GeV}$. The solid, dashed and small dashed lines represent the BR without extra dimension, including a single extra dimension for $1/R = 500\,\mathrm{GeV}$ and including two extra dimensions for $1/R = 500\,\mathrm{GeV}$. Similar to the previous process, BR is not sensitive to the extra dimension effects for a single extra dimension and this sensitivity increases for two NUED. The enhancement of the BR of two NUED is more than two orders larger compared to the one without extra dimensions. Figure 6 is devoted to the compactification scale $1/R$ dependence of BR for $\bar{\xi}^{D}_{\mathrm{N},\tau\tau} = 10\,\mathrm{GeV}$, $\bar{\xi}^{D}_{\mathrm{N},\tau\mu} = 1\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$ and $m_{A^0} = 200\,\mathrm{GeV}$. In this figure the solid, dashed and small dashed lines represent the BR without extra dimension, including a single extra dimension and including two extra dimensions. The enhancement in BR for the intermediate values of the compactification scale, namely $1/R \sim 1000\,\mathrm{GeV}$, is more than one order. Similar to the previous decay, this enhancement becomes small for the larger values of the compactification scales, $1/R > 5000\,\mathrm{GeV}$.

Finally, Fig. 7, see (8), is devoted to $\bar{\xi}^{D}_{\mathrm{N},\tau\tau}$ (the compactification scale $1/R$) dependence of the BR of the decay $Z \to \tau^\pm \mu^\pm$ for $\bar{\xi}^{D}_{\mathrm{N},\tau\mu} = 1\,\mathrm{GeV}$ ($\bar{\xi}^{D}_{\mathrm{N},\tau\mu} = 1\,\mathrm{GeV}$,

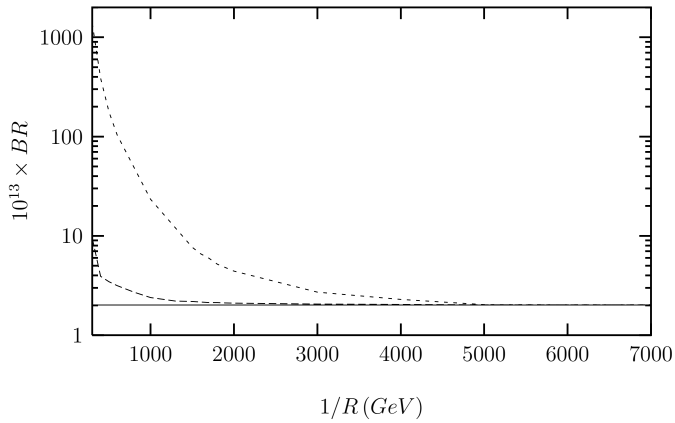


Fig. 4. The compactification scale $1/R$ dependence of $\mathrm{BR}\,(Z \to \mu^\pm e^\pm)$ for $\bar{\xi}^{D}_{\mathrm{N},\tau e} = 0.05\,\mathrm{GeV}$, $\bar{\xi}^{D}_{\mathrm{N},\tau\mu} = 1\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$ and $m_{A^0} = 200\,\mathrm{GeV}$. The solid, dashed and small dashed lines represent the BR without extra dimension, including a single extra dimension and including two extra dimensions

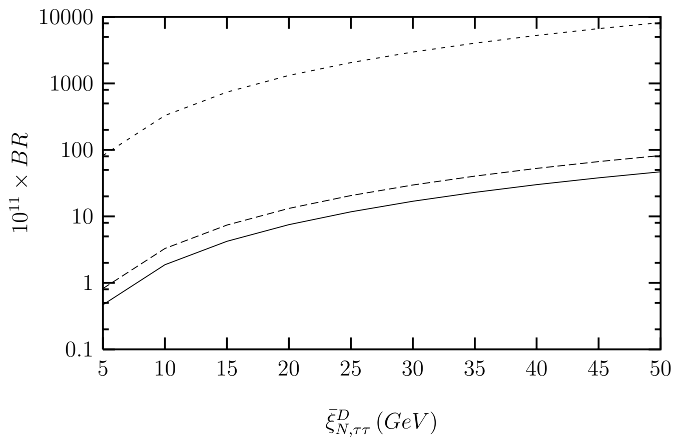


Fig. 5. $\bar{\xi}^{D}_{\mathrm{N},\tau\tau}$ dependence of the $\mathrm{BR}\,(Z \to \tau^\pm e^\pm)$ for $\bar{\xi}^{D}_{\mathrm{N},\tau e} = 0.05\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$ and $m_{A^0} = 200\,\mathrm{GeV}$. The solid, dashed and small dashed lines represent the BR without extra dimension, including a single extra dimension for $1/R = 500\,\mathrm{GeV}$ and including two extra dimensions for $1/R = 500\,\mathrm{GeV}$

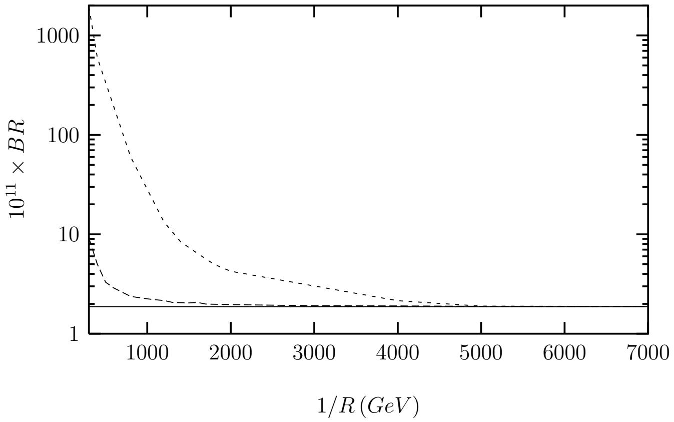


Fig. 6. The compactification scale $1/R$ dependence of the BR$\,(Z \to \tau^{\pm}\, e^{\pm})$ for $\bar{\xi}^{D}_{\mathrm{N},\tau\tau} = 10\,\mathrm{GeV}$, $\bar{\xi}^{D}_{\mathrm{N},\tau\mu} = 1\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$ and $m_{A^0} = 200\,\mathrm{GeV}$. The solid, dashed and small dashed lines represent the BR without extra dimension, including a single extra dimension and including two extra dimensions

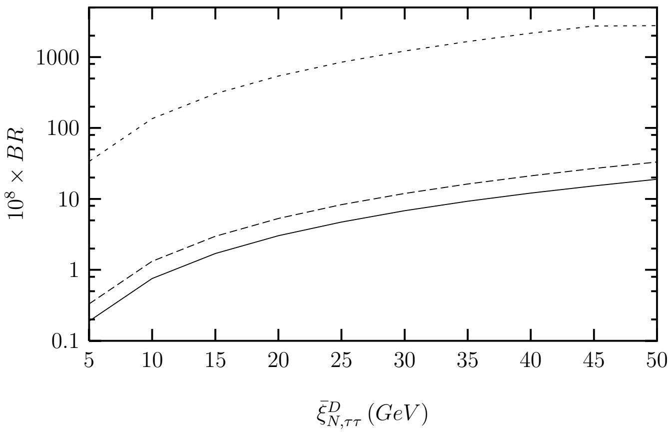


Fig. 7. The $\bar{\xi}^{D}_{\mathrm{N},\tau\tau}$ dependence of the BR of the two decay $Z \to \tau^{\pm}\, \mu^{\pm}$ for $\bar{\xi}^{D}_{\mathrm{N},\tau\mu} = 1\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$ and $m_{A^0} = 200\,\mathrm{GeV}$. The solid, dashed and small dashed lines represent the BR without extra dimension, including a single extra dimension for $1/R = 500\,\mathrm{GeV}$ and including two extra dimensions for $1/R = 500\,\mathrm{GeV}$

$\bar{\xi}^{D}_{\mathrm{N},\tau\tau} = 10\,\mathrm{GeV}$), $m_{h^0} = 100\,\mathrm{GeV}$ and $m_{A^0} = 200\,\mathrm{GeV}$. In Fig. 7 the solid, dashed and small dashed lines represent the BR without extra dimension, including a single extra dimension for $1/R = 500\,\mathrm{GeV}$ and including two extra dimensions for $1/R = 500\,\mathrm{GeV}$. In the case of two extra dimensions, even for small Yukawa couplings, it is possible to reach the experimental upper limit of the BR of the corresponding decay, since the enhancement in the BR is two orders larger compared to the case without extra dimensions. In Fig. 8, the solid, dashed and small dashed lines represent the BR without extra dimension, including a single extra dimension and including two extra dimensions. It is observed that, in the case of two extra dimensions, the BR reaches almost twice the one without extra dimensions, even for intermediate values of the compactification scale, $1/R = 2000\,\mathrm{GeV}$. For the larger values of the compactification scales, $1/R > 5000\,\mathrm{GeV}$, there is no enhancement in the BR of the present decay.

At this stage we would like to present our results briefly.

(1) With the inclusion of a single NUED, the enhancement in the BR of the LFV Z decays is small for the intermediate values of the compactification scale $1/R$.

(2) In the case of two NUED, even for the small values of the Yukawa couplings, it is possible to reach the experimental upper limits of the BRs of the LFV Z decays, since the enhancement in the BR is two orders larger compared to the case without extra dimensions for the intermediate values of the compactification scale $1/R$. This enhancement is due to the crowd of the KK modes and it is an interesting result which may ensure important information to test the existence of the NUED, and if it exists, to decide its number and to predict the lower limit of the compactification scale, with the help of more accurate experimental results.

As a summary, the effect of two NUED on the BRs of LFV Z decays $Z \to l_1^{\pm}\, l_2^{\pm}$ is strong and the more accurate future experimental results of these decays will be useful to test the possible signals coming from the extra dimensions.

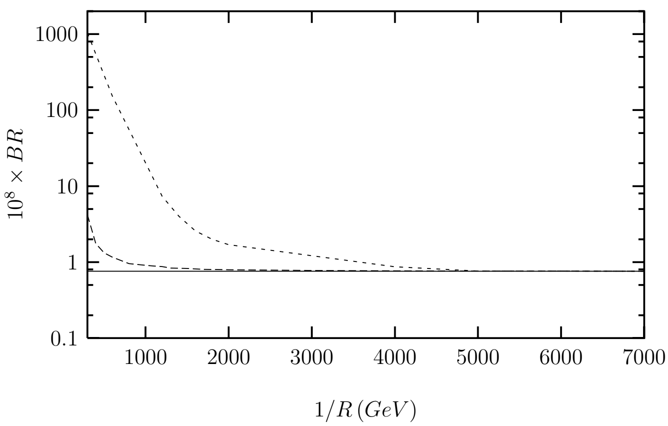


Fig. 8. The compactification scale $1/R$ dependence of the BR of the two decay $Z \to \tau^{\pm}\, \mu^{\pm}$ for $\bar{\xi}^{D}_{\mathrm{N},\tau\mu} = 1\,\mathrm{GeV}$, $\bar{\xi}^{D}_{\mathrm{N},\tau\tau} = 10\,\mathrm{GeV}$, $m_{h^0} = 100\,\mathrm{GeV}$ and $m_{A^0} = 200\,\mathrm{GeV}$. The solid, dashed and small dashed lines represent the BR without extra dimension, including a single extra dimension and including two extra dimensions

Acknowledgements. This work has been supported by the Turkish Academy of Sciences in the framework of the Young Scientist Award Program. (EOI-TUBA-GEBIP/2001-1-8)

Appendix A: The explicit expressions appearing in the text

Here we present the explicit expressions for f_V^0, f_A^0, f_M^0 and f_E^0 [8] (see (12)):

$$f_V^0 = \frac{g}{64\,\pi^2\,\cos\theta_{\mathrm{W}}}$$

$$
\begin{aligned}
&\times \int_0^1 \mathrm{d}x \, \frac{1}{m^2_{l_2^+} - m^2_{l_1^-}} \Big\{ c_V \, (m_{l_2^+} + m_{l_1^-}) \\
&\times \Bigg((-m_i \, \eta_i^+ + m_{l_1^-} (-1+x) \, \eta_i^V) \, \ln \frac{L^{\mathrm{self}}_{1,\, h^0}}{\mu^2} \\
&+ (m_i \, \eta_i^+ - m_{l_2^+} (-1+x) \, \eta_i^V) \, \ln \frac{L^{\mathrm{self}}_{2,\, h^0}}{\mu^2} \\
&+ (m_i \, \eta_i^+ + m_{l_1^-} (-1+x) \, \eta_i^V) \, \ln \frac{L^{\mathrm{self}}_{1,\, A^0}}{\mu^2} \\
&- (m_i \, \eta_i^+ + m_{l_2^+} (-1+x) \, \eta_i^V) \, \ln \frac{L^{\mathrm{self}}_{2,\, A^0}}{\mu^2} \Bigg) \\
&+ c_A \, (m_{l_2^+} - m_{l_1^-}) \\
&\times \Bigg((-m_i \, \eta_i^- + m_{l_1^-} (-1+x) \, \eta_i^A) \, \ln \frac{L^{\mathrm{self}}_{1,\, h^0}}{\mu^2} \\
&+ (m_i \, \eta_i^- + m_{l_2^+} (-1+x) \, \eta_i^A) \, \ln \frac{L^{\mathrm{self}}_{2,\, h^0}}{\mu^2} \\
&+ (m_i \, \eta_i^- + m_{l_1^-} (-1+x) \, \eta_i^A) \, \ln \frac{L^{\mathrm{self}}_{1,\, A^0}}{\mu^2} \\
&+ (-m_i \, \eta_i^- + m_{l_2^+} (-1+x) \, \eta_i^A) \, \ln \frac{L^{\mathrm{self}}_{2,\, A^0}}{\mu^2} \Bigg) \Big\} \\
&- \frac{g}{64 \, \pi^2 \, \cos \theta_{\mathrm{W}}} \\
&\times \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \\
&\times \Bigg\{ m_i^2 \, (c_A \, \eta_i^A - c_V \, \eta_i^V) \left(\frac{1}{L^{\mathrm{ver}}_{A^0}} + \frac{1}{L^{\mathrm{ver}}_{h^0}} \right) \\
&- (1-x-y) \\
&\times m_i \Bigg(c_A \, (m_{l_2^+} - m_{l_1^-}) \, \eta_i^- \left(\frac{1}{L^{\mathrm{ver}}_{h^0}} - \frac{1}{L^{\mathrm{ver}}_{A^0}} \right) \\
&+ c_V \, (m_{l_2^+} + m_{l_1^-}) \, \eta_i^+ \left(\frac{1}{L^{\mathrm{ver}}_{h^0}} + \frac{1}{L^{\mathrm{ver}}_{A^0}} \right) \Bigg) \\
&- (c_A \, \eta_i^A + c_V \, \eta_i^V) \\
&\times \Bigg(-2 + (q^2 \, x \, y + m_{l_1^-} \, m_{l_2^+} \, (-1+x+y)^2) \\
&\times \left(\frac{1}{L^{\mathrm{ver}}_{h^0}} + \frac{1}{L^{\mathrm{ver}}_{A^0}} \right) \\
&- \ln \frac{L^{\mathrm{ver}}_{h^0}}{\mu^2} \, \frac{L^{\mathrm{ver}}_{A^0}}{\mu^2} \Bigg) \\
&- (m_{l_2^+} + m_{l_1^-}) \, (1-x-y) \\
&\times \Bigg(\frac{\eta_i^A \, (x \, m_{l_1^-} + y \, m_{l_2^+}) + m_i \, \eta_i^-}{2 \, L^{\mathrm{ver}}_{A^0 \, h^0}} \\
&+ \frac{\eta_i^A \, (x \, m_{l_1^-} + y \, m_{l_2^+}) - m_i \, \eta_i^-}{2 \, L^{\mathrm{ver}}_{h^0 \, A^0}} \Bigg) \\
&+ \frac{1}{2} \eta_i^A \, \ln \frac{L^{\mathrm{ver}}_{A^0 \, h^0}}{\mu^2} \, \frac{L^{\mathrm{ver}}_{h^0 \, A^0}}{\mu^2} \Bigg\} \, ,
\end{aligned}
$$

$$
\begin{aligned}
f_A^0 &= \frac{-g}{64 \, \pi^2 \, \cos \theta_{\mathrm{W}}} \\
&\times \int_0^1 \mathrm{d}x \, \frac{1}{m^2_{l_2^+} - m^2_{l_1^-}} \Big\{ c_V \, (m_{l_2^+} - m_{l_1^-}) \\
&\times \Bigg((m_i \, \eta_i^- + m_{l_1^-} (-1+x) \, \eta_i^A) \, \ln \frac{L^{\mathrm{self}}_{1,\, A^0}}{\mu^2} \\
&+ (-m_i \, \eta_i^- + m_{l_2^+} (-1+x) \, \eta_i^A) \, \ln \frac{L^{\mathrm{self}}_{2,\, A^0}}{\mu^2} \\
&+ (-m_i \, \eta_i^- + m_{l_1^-} (-1+x) \, \eta_i^A) \, \ln \frac{L^{\mathrm{self}}_{1,\, h^0}}{\mu^2} \\
&+ (m_i \, \eta_i^- + m_{l_2^+} (-1+x) \, \eta_i^A) \, \ln \frac{L^{\mathrm{self}}_{2,\, h^0}}{\mu^2} \Bigg) \\
&+ c_A \, (m_{l_2^+} + m_{l_1^-}) \\
&\times \Bigg((m_i \, \eta_i^+ + m_{l_1^-} (-1+x) \, \eta_i^V) \, \ln \frac{L^{\mathrm{self}}_{1,\, A^0}}{\mu^2} \\
&- (m_i \, \eta_i^+ + m_{l_2^+} (-1+x) \, \eta_i^V) \, \ln \frac{L^{\mathrm{self}}_{2,\, A^0}}{\mu^2} \\
&+ (-m_i \, \eta_i^+ + m_{l_1^-} (-1+x) \, \eta_i^V) \, \ln \frac{L^{\mathrm{self}}_{1,\, h^0}}{\mu^2} \\
&+ (m_i \, \eta_i^+ - m_{l_2^+} (-1+x) \, \eta_i^V) \, \frac{\ln L^{\mathrm{self}}_{2,\, h^0}}{\mu^2} \Bigg) \Big\} \\
&+ \frac{g}{64 \, \pi^2 \, \cos \theta_{\mathrm{W}}} \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \\
&\times \Bigg\{ m_i^2 \, (c_V \, \eta_i^A - c_A \, \eta_i^V) \left(\frac{1}{L^{\mathrm{ver}}_{A^0}} + \frac{1}{L^{\mathrm{ver}}_{h^0}} \right) \\
&- m_i \, (1-x-y) \\
&\times \left(c_V \, (m_{l_2^+} - m_{l_1^-}) \, \eta_i^- + c_A \, (m_{l_2^+} + m_{l_1^-}) \, \eta_i^+ \right) \\
&\times \left(\frac{1}{L^{\mathrm{ver}}_{h^0}} - \frac{1}{L^{\mathrm{ver}}_{A^0}} \right) \\
&+ (c_V \, \eta_i^A + c_A \, \eta_i^V) \\
&\times \Bigg(-2 + (q^2 \, x \, y - m_{l_1^-} \, m_{l_2^+} \, (-1+x+y)^2) \\
&\times \left(\frac{1}{L^{\mathrm{ver}}_{h^0}} + \frac{1}{L^{\mathrm{ver}}_{A^0}} \right) \\
&- \ln \frac{L^{\mathrm{ver}}_{h^0}}{\mu^2} \, \frac{L^{\mathrm{ver}}_{A^0}}{\mu^2} \Bigg) \\
&- (m_{l_2^+} - m_{l_1^-}) \, (1-x-y) \\
&\times \Bigg(\frac{\eta_i^V \, (x \, m_{l_1^-} - y \, m_{l_2^+}) + m_i \, \eta_i^+}{2 \, L^{\mathrm{ver}}_{A^0 \, h^0}} \\
&+ \frac{\eta_i^V \, (x \, m_{l_1^-} - y \, m_{l_2^+}) - m_i \, \eta_i^+}{2 \, L^{\mathrm{ver}}_{h^0 \, A^0}} \Bigg) \\
&- \frac{1}{2} \eta_i^V \, \ln \frac{L^{\mathrm{ver}}_{A^0 \, h^0}}{\mu^2} \, \frac{L^{\mathrm{ver}}_{h^0 \, A^0}}{\mu^2} \Bigg\} \, ,
\end{aligned}
$$

$$
\begin{aligned}
f_M^0 = &-\frac{g\, m_W}{64\,\pi^2\,\cos\theta_W}\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y \\
&\times\Bigg\{\Big((1-x-y)\,(c_V\,\eta_i^V + c_A\,\eta_i^A)\,(x\,m_{l_1^-}+y\,m_{l_2^+}) \\
&+ m_i\,(c_A\,(x-y)\,\eta_i^- + c_V\,\eta_i^+\,(x+y))\Big)\,\frac{1}{L^{\mathrm{ver}}_{h^0}} \\
&+\Big((1-x-y)\,(c_V\,\eta_i^V + c_A\,\eta_i^A)\,(x\,m_{l_1^-}+y\,m_{l_2^+}) \\
&- m_i\,(c_A\,(x-y)\,\eta_i^- + c_V\,\eta_i^+\,(x+y))\Big)\,\frac{1}{L^{\mathrm{ver}}_{A^0}} \\
&-(1-x-y) \\
&\times\left(\frac{\eta_i^A\,(x\,m_{l_1^-}+y\,m_{l_2^+})}{2}\left(\frac{1}{L^{\mathrm{ver}}_{A^0\,h^0}}+\frac{1}{L^{\mathrm{ver}}_{h^0\,A^0}}\right)\right. \\
&\left.+\frac{m_i\,\eta_i^-}{2}\left(\frac{1}{L^{\mathrm{ver}}_{h^0\,A^0}}-\frac{1}{L^{\mathrm{ver}}_{A^0\,h^0}}\right)\right)\Bigg\}\,, \\
f_E^0 = &-\frac{g\, m_W}{64\,\pi^2\,\cos\theta_W}\int_0^1 \mathrm{d}x \int_0^{1-x}\mathrm{d}y \\
&\times\{((1-x-y) \\
&\times\Big(-(c_V\,\eta_i^A + c_A\,\eta_i^V)\,(x\,m_{l_1^-}-y\,m_{l_2^+})\Big) \\
&- m_i\,(c_A\,(x-y)\,\eta_i^+ + c_V\,\eta_i^-\,(x+y)))\,\frac{1}{L^{\mathrm{ver}}_{h^0}} \\
&+((1-x-y) \\
&\times\Big(-(c_V\,\eta_i^A + c_A\,\eta_i^V)\,(x\,m_{l_1^-}-y\,m_{l_2^+})\Big) \\
&+ m_i\,(c_A\,(x-y)\,\eta_i^+ + c_V\,\eta_i^-\,(x+y)))\,\frac{1}{L^{\mathrm{ver}}_{A^0}} \\
&+(1-x-y) \\
&\times\left(\frac{\eta_i^V}{2}\,(m_{l_1^-}\,x-m_{l_2^+}\,y)\left(\frac{1}{L^{\mathrm{ver}}_{A^0\,h^0}}+\frac{1}{L^{\mathrm{ver}}_{h^0\,A^0}}\right)\right. \\
&\left.+\frac{m_i\,\eta_i^+}{2}\left(\frac{1}{L^{\mathrm{ver}}_{A^0\,h^0}}-\frac{1}{L^{\mathrm{ver}}_{h^0\,A^0}}\right)\right)\Bigg\}\,,
\end{aligned}
\tag{A.1}
$$

where

$$
\begin{aligned}
L^{\mathrm{self}}_{1,\,h^0} &= m_{h^0}^2\,(1-x) + (m_i^2 - m_{l_1^-}^2\,(1-x))\,x\,, \\
L^{\mathrm{self}}_{1,\,A^0} &= L^{\mathrm{self}}_{1,\,h^0}(m_{h^0}\to m_{A^0})\,, \\
L^{\mathrm{self}}_{2,\,h^0} &= L^{\mathrm{self}}_{1,\,h^0}(m_{l_1^-}\to m_{l_2^+})\,, \\
L^{\mathrm{self}}_{2,\,A^0} &= L^{\mathrm{self}}_{1,\,A^0}(m_{l_1^-}\to m_{l_2^+})\,, \\
L^{\mathrm{ver}}_{h^0} &= m_{h^0}^2\,(1-x-y) + m_i^2\,(x+y) - q^2\,x\,y\,, \\
L^{\mathrm{ver}}_{h^0\,A^0} &= m_{A^0}^2\,x + m_i^2\,(1-x-y) + (m_{h^0}^2 - q^2\,x)\,y\,, \\
L^{\mathrm{ver}}_{A^0} &= L^{\mathrm{ver}}_{h^0}(m_{h^0}\to m_{A^0})\,, \\
L^{\mathrm{ver}}_{A^0\,h^0} &= L^{\mathrm{ver}}_{h^0\,A^0}(m_{h^0}\to m_{A^0})\,,
\end{aligned}
\tag{A.2}
$$

and

$$
\begin{aligned}
\eta_i^V &= \xi^D_{\mathrm{N},l_1 i}\xi^{D\,*}_{\mathrm{N},i l_2} + \xi^{D\,*}_{\mathrm{N},i l_1}\xi^D_{\mathrm{N},l_2 i}\,, \\
\eta_i^A &= \xi^D_{\mathrm{N},l_1 i}\xi^{D\,*}_{\mathrm{N},i l_2} - \xi^{D\,*}_{\mathrm{N},i l_1}\xi^D_{\mathrm{N},l_2 i}\,, \\
\eta_i^+ &= \xi^{D\,*}_{\mathrm{N},i l_1}\xi^{D\,*}_{\mathrm{N},i l_2} + \xi^D_{\mathrm{N},l_1 i}\xi^D_{\mathrm{N},l_2 i}\,, \\
\eta_i^- &= \xi^{D\,*}_{\mathrm{N},i l_1}\xi^{D\,*}_{\mathrm{N},i l_2} - \xi^D_{\mathrm{N},l_1 i}\xi^D_{\mathrm{N},l_2 i}\,.
\end{aligned}
\tag{A.3}
$$

The parameters c_V and c_A are $c_A = -\frac{1}{4}$ and $c_V = \frac{1}{4} - \sin^2\theta_W$. In (A.3) the flavor changing couplings $\bar{\xi}^D_{\mathrm{N},l_j i}$ represent the effective interaction between the internal lepton i ($i = e, \mu, \tau$) and the outgoing (incoming) $j = 1$ ($j = 2$) one. Here the couplings $\bar{\xi}^D_{\mathrm{N},l_j i}$ are complex in general and they can be parametrized as

$$
\xi^D_{\mathrm{N},i l_j} = |\xi^D_{\mathrm{N},i l_j}|\,\mathrm{e}^{\mathrm{i}\theta_{ij}}\,, \tag{A.4}
$$

where i, l_j denote the lepton flavors and θ_{ij} are CP violating parameters which are the possible sources of the lepton EDM. However, in the present work we take these couplings real.

Appendix B: Gauge boson mass matrix and gauge coupling

Here we study an abelian model in the case of a single extra dimension (two extra dimensions) with two Higgs fields, where one Higgs field, $\phi_1(x)$, is localized at the $y = 0$ ($y = z = 0$) boundary of the S^1/Z_2 ($(S^1 \times S^1)/Z_2$) orbifold and the other one, $\phi_1(x, y)$ ($\phi_1(x, y, z)$), is accessible to the extra dimension(s). Furthermore, we choose the case that only the first Higgs field has a non-zero vacuum expectation value, and, including a single extra dimension, the Higgs fields read

$$
\begin{aligned}
\phi_1(x) &= \frac{1}{\sqrt{2}}\,(v + h_1(x) + \mathrm{i}\,\chi_1(x)) \\
\phi_2(x) &= \frac{1}{\sqrt{2}}\,(h_2(x,y) + \mathrm{i}\,\chi_2(x,y))\,.
\end{aligned}
\tag{B.1}
$$

Our aim is to obtain the gauge boson mass matrix, which is not diagonal in the case of the non-universal extra dimensions where the gauge sector and the Higgs field ϕ_2 is accessible to the extra dimensions but the first Higgs field is not. In the case of a single extra dimension, the detailed analysis of this issue has been done in [31] and both Higgs fields are assumed to have vacuum expectation values in that work. We will present the crucial steps of this work briefly and we repeat the same analysis for two extra dimensions.

The part of the Lagrangian which carries the gauge and Higgs sector in a single extra dimension reads

$$
\begin{aligned}
\mathcal{L}(x,y) = &-\frac{1}{4}\,F^{MN}\,F_{MN} + (D_M\,\phi_2)^*\,(D^M\,\phi_2) \\
&+\delta(y)\,(D_\mu\,\phi_1)^*\,(D^\mu\,\phi_1) \\
&- V(\phi_1,\phi_2) + \mathcal{L}_{\mathrm{GF}}(x,y)\,,
\end{aligned}
\tag{B.2}
$$

where V is the CP and gauge invariant Higgs potential, $D_M = \partial_M + \mathrm{i}e_5 A_M(x,y)$ ($M = \mu, 5$) is the covariant derivative in five dimensions and $\mathcal{L}_{\mathrm{GF}}(x,y)$ is the gauge fixing term.

Now, we will present the sources of the gauge boson mass matrix in the Lagrangian (B.2):
(1) $F^{5\mu}\,F_{5\mu}$ in the part $F^{MN}\,F_{MN}$,
(2) $(A_\mu\,A^\mu)$ in the part $(D_\mu\,\phi_1)^*\,(D^\mu\,\phi_1)$, where $F^{MN} =$

$\partial^N A^N - \partial^M A^N$, and, after compactification, the gauge fields A^N read

$$A_\mu(x,y) = \frac{1}{(2\pi R)^{1/2}} \left\{ A_\mu^{(0)}(x) + 2^{1/2} \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \cos(ny/R) \right\}, \tag{B.3}$$
$$A_5(x,y) = \frac{1}{(2\pi R)^{1/2}} \left\{ 2^{1/2} \sum_{n=1}^{\infty} A_5^{(n)}(x) \sin(ny/R) \right\}.$$

Notice that the $(D_M\,\phi_2)^*\,(D^M\,\phi_2)$ term in the Lagrangian (see (B.2)) does not produce any mass term for the gauge field since the scalar field ϕ_2 does not have any vacuum expectation value. The integration over the extra dimension y results in the mixing of zero mode and KK mode gauge fields and the gauge boson mass matrix is obtained as [31]:

$$M_A^2 = \begin{pmatrix} m^2 & \sqrt{2}\,m^2 & \sqrt{2}\,m^2 & \cdots \\ \sqrt{2}\,m^2 & 2\,m^2 + (1/R)^2 & 2\,m^2 & \cdots \\ \sqrt{2}\,m^2 & 2\,m^2 & 2\,m^2 + (2/R)^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{B.4}$$

where $m^2 = e^2 v^2$. Using the determinant equation

$$\det\left(M_A^2 - \lambda\, I\right) = \left(\prod_{n=1}^{\infty} (n^2/R^2 - \lambda) \right) \times \left(m^2 - \lambda - 2\,\lambda\, m^2 \sum_{n=1}^{\infty} \frac{1}{(n/R)^2 - \lambda} \right) = 0, \tag{B.5}$$

the eigenvalues of the matrix are obtained by solving the transcendental equation

$$m_{A^{(n)}} = \pi\, m^2\, R\, \cot(\pi\, m_{A^{(n)}}\, R), \tag{B.6}$$

and the KK mass eigenstates $\hat{A}_\mu^{(n)}$ are given by

$$\hat{A}_\mu^{(n)} = \left(1 + \pi^2\, m^2\, R^2 + \frac{m^2_{A^{(n)}}}{m^2} \right)^{-1/2} \times \sum_{j=0}^{\infty} \frac{2\, m_{A^{(n)}}\, m}{m^2_{A^{(n)}} - (j/R)^2} \left(\frac{1}{\sqrt{2}} \right)^{\delta_{j,0}} A_\mu(j). \tag{B.7}$$

For the non-abelian case the gauge field mass spectrum is analogous to the abelian one presented above and the transcendental equation for the Z boson is

$$m_{Z^{(n)}} = \pi\, m_Z^2\, R\, \cot(\pi\, m_{Z^{(n)}}\, R), \tag{B.8}$$

and the corresponding coupling reads

$$g_{Z^{(n)}} = \sqrt{2}\, g \left(1 + \frac{m_Z^2}{m^2_{Z^{(n)}}} + \frac{\pi^2\, R^2\, m_Z^4}{m^2_{Z^{(n)}}} \right)^{-1/2}. \tag{B.9}$$

At this stage we try to make the same analysis for two extra dimensions. The part of the Lagrangian which carries the gauge and Higgs sector in two extra dimensions is

$$\mathcal{L}(x,y) = -\frac{1}{4}\, F^{MN}\, F_{MN} + (D_M\,\phi_2)^*\,(D^M\,\phi_2) + \delta(y)\,\delta(z)\,(D_\mu\,\phi_1)^*\,(D^\mu\,\phi_1) - V(\phi_1,\phi_2) + \mathcal{L}_{\mathrm{GF}}(x,y,z), \tag{B.10}$$

$D_M = \partial_M + \mathrm{i}e_6 A_M(x,y,z)$ $(M = \mu, 5, 6)$ is the covariant derivative in six dimensions. In this case the sources of the gauge boson mass matrix in the Lagrangian (see (B.10)) are
(1) $F^{5\mu}\, F_{5\mu}$ and $F^{6\mu}\, F_{6\mu}$ in the part $F^{MN}\, F_{MN}$,
(2) $(A_\mu\, A^\mu)$ in the part $(D_\mu\,\phi_1)^*\,(D^\mu\,\phi_1)$, and, after compactification, the gauge fields A_N read

$$A_\mu(x,y) = \frac{1}{(2\pi R)} \left\{ A_\mu^{(0,0)}(x) + 2 \sum_{n,r}^{\infty} A_\mu^{(n,r)}(x) \cos(ny/R + rz/R) \right\},$$
$$A_{5\,(6)}(x,y) = \frac{1}{(2\pi R)} \left\{ 2 \sum_{n,r}^{\infty} A_{5\,(6)}^{(n,r)}(x) \sin(ny/R + rz/R) \right\}. \tag{B.11}$$

The integration over the extra dimensions y and z results in the mixing of zero mode and KK mode gauge fields similar to the one extra dimension case and the gauge boson mass matrix is obtained as

$$M_A^2 = \begin{pmatrix} m^2 & 2\,m^2 & 2\,m^2 & 2\,m^2 & 2\,m^2 & \cdots \\ 2\,m^2 & 4\,m^2 + 2/R^2 & 4\,m^2 & 4\,m^2 & 4\,m^2 & \cdots \\ 2\,m^2 & 4\,m^2 & 4\,m^2 + 2/R^2 & 4\,m^2 & 4\,m^2 & \cdots \\ 2\,m^2 & 4\,m^2 & 4\,m^2 & 4\,m^2 + 4/R^2 & 4\,m^2 & \cdots \\ 2\,m^2 & 4\,m^2 & 4\,m^2 & 4\,m^2 & 4\,m^2 + 10/R^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \tag{B.12}$$

with $m^2 = e^2 v^2$. Here, the mass spectrum is richer compared to the single extra dimension case. Now the determinant equation reads

$$\det\left(M_A^2 - \lambda\, I\right) = \left(\prod_{n,r}^{\infty} \left(\frac{2\,(n^2 + r^2)}{R^2} - \lambda \right) \right) \tag{B.13}$$

$$\times \left(m^2 - \lambda - 4\,\lambda\, m^2 \sum_{n,r}^{\infty} \frac{1}{\frac{2\,(n^2+r^2)}{R^2} - \lambda} \right) = 0\,,$$

where the indices n and r are all positive integers including zero, but both are not zero at the same time, and the transcendental equation to obtain the eigenvalues of the matrix is

$$\begin{aligned} & m^4_{A^{(p)}} \\ & = -\left(\frac{2\,m}{R}\right)^2 + \left(\frac{2}{R^2} + 10\,m^2\right)\, m^2_{A^{(p)}} \\ & - (m_{A^{(p)}}\, m^2\, \pi\, R') \left(\frac{1}{R'^2} - m^2_{A^{(p)}}\right)\, \cot\left(\pi\, m_{A^{(p)}}\, R'\right) \\ & - (m^2_{A^{(p)}}\, m^2) \left(\frac{1}{R'^2} - m^2_{A^{(p)}}\right) \sum_{r=1}^{\infty} \frac{\pi\, R'}{\sqrt{\lambda_r}}\, \cot\left(\pi\, \sqrt{\lambda_r}\, R'\right)\,, \end{aligned} \tag{B.14}$$

where $\lambda_r = m^2_{A^{(p)}} - r^2/R'^2$, $R' = R/\sqrt{2}$ and p is a positive integer. Similar to the case of one extra dimension, for the non-abelian case, the gauge field mass spectrum is analogous to the abelian one presented above and the transcendental equation is obtained by replacing the mass $m_{A^{(n)}}$ in (B.14) by $m_{Z^{(n)}}$. For two extra dimensions there appears a new gauge coupling due to the complicated mass mixing, however, in our numerical calculations, we used the one obtained in the single extra dimension case by expecting that the new contributions do not affect the behaviors of the physical parameters we study. Notice that this coupling enters in the calculations only for the zero mode Z boson case, since there is no diagram which includes the KK mode virtual Z bosons, namely

$$g_Z = \sqrt{2}\, g \left(1 + \frac{m_Z^2}{m^2_{Z^{(0)}}} + \frac{\pi^2\, R^2\, m_Z^4}{m^2_{Z^{(0)}}}\right)^{-1/2}\,, \tag{B.15}$$

where $m_{Z^{(0)}}$ is obtained by solving (B.8) for $n = 0$.

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